Q. P. Code : 38400

Duration: 3 Hrs **Maximum marks: 70**

Note: **All** Questions are **compulsory**.
Use of **simple calculator** is **allowed**.
Figure at right indicate **maximum marks**.

Q1. (a) Attempt any 7 [2 marks each]: [14]

(i) If A = $\begin{bmatrix} 4 & -2 \\ -5 & 7 \end{bmatrix}$ B = $\begin{bmatrix} -3 & 6 \\ 4 & 3 \end{bmatrix}$ then $(2A + B)^T$ is :

(a) $\begin{bmatrix} 5 & 2 \\ -6 & 17 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -5 \\ -2 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & -6 \\ 2 & 17 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 4 \\ 6 & -3 \end{bmatrix}$

(ii) The N^{th} derivative of $y = 2\cos^2 x$ is :
(a) $-2^n \cos(2x+n\pi/2)$ (b) $-4 \cos x \sin x$ (c) $2^n \cos(2x+n\pi/2)$ (d) $2^n \sin(2x+n\pi/2)$

(iii) For $f(x, y) = x^2 + xy + y^2$, the value of $\frac{\partial^2 f}{\partial x \partial y}$ is:
(a) 2x + y (b) 1 (c) 2 (d) x + 2y

(iv) $\Delta f(x)$ for the function $f(x) = 1/x$, by taking h =1 is :
(a) $-1/x^2$ (b) $1/x^2$ (c) $-1/(x^2 + x)$ (d) $1/(x^2 + x)$

(v) The volume of the solid obtained by the revolution of area y = sinx and x-axis between the interval 0 to π is :

(a) 2π (b) $\pi^2/2$ (c) $\pi^2/4$ (d) π/2

(vi) The solution of the differential equation xdx + ydy = 0 is:
(a) $x^2 + y^2 = c$ (b) $x^2 - y^2 = c$ (c) x + y = c (d) x - y = c

(vii) The value of $\int \log x \, dx$ is :

(a) xlogx – 1 + c (b) xlogx + 1 -c (c) x(logx + 1) + c (d) x(logx – 1) + c

(viii) The differential equation for the function y = mx is:

(a) x dy – y dx = 0 (b) x dy + y dx = 0 (c) y dy – x dx = 0 (d) y dy + x dx = 0

(ix) The inverse of the matrix A=$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ is:
(a) $\frac{1}{14}\begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$ (b) $\frac{1}{12}\begin{bmatrix} -3 & 4 \\ -2 & 2 \end{bmatrix}$ (c) $\frac{1}{14}\begin{bmatrix} 2 & -4 \\ -2 & 3 \end{bmatrix}$ (d) $\frac{1}{14}\begin{bmatrix} 2 & 2 \\ -4 & 3 \end{bmatrix}$

(b) Attempt any 1: [1]

(x) The value of $\int_{-2}^{2} \frac{x}{1+x^2} dx$ is: (a) -2 (b) 2 (c) 0 (d) 4

(xi) N^{th} derivative of y = $\frac{1}{9x+2}$ is (a) $\frac{(-1)^{n-1}(n-1)!9^n}{(9x+2)^n}$ (b) $\frac{(-1)^n(n)!9^n}{(9x+2)^{n+1}}$
(c) $\frac{(-1)^{n-1}(n-1)!9^n}{(9x+2)^{n+1}}$ (d) $\frac{(-1)^n(n)!9^n}{(9x+2)^n}$

TURN OVER

Q2. (a) Attempt any two (4 marks each) **[8]**

(i) Find the Nth derivative of $y=\frac{x}{(x+3)(x-2)}$

(ii) Using Taylor's series, expand sin x in ascending powers of $(x-\frac{\pi}{2})$

(iii) If U = y sin(xy), prove that $y\frac{\partial U}{\partial y} - x\frac{\partial U}{\partial x} = U$

(b) Attempt any one(3 marks) **[3]**

(i) Verify Rolle's theorem for the function $f(x)=x^2-3x+2$ in [1,2]

(ii) If $y=x^3\log x$, find: y_4 using Leibnitz's theorem.

Q3. (a) Attempt any two (4 marks each) **[8]**

(i) Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} Sin^n\ x\ dx$, hence evaluate $\int_0^{\frac{\pi}{2}} Sin^{10}\ x\ dx$

(ii) Find the whole area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(iii) Prove that $\int_0^{\frac{\pi}{2}} cos^2 x\ dx = \frac{\pi}{4}$

(b) Attempt any one(3 marks) **[3]**

(i) Find the area bounded by the parabola $x^2=4y$, X-axis and the lines x=1 and x=3

(ii) By using the properties of Definite Integral, Evaluate $I=\int_0^2 \left(\frac{x^2-4}{x^2+4}\right) dx$

Q4. (a) Attempt any two (4 marks each) **[8]**

(i) By using the Adjoint method, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

(ii) Prove that $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ = (x-y)(y-z)(z-x)

(iii) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(b) Attempt any one(3 marks) **[3]**

(i) Find the rank of the matrix $A=\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

(ii) Solve by Cramer's rule:-
x+y+z=6; 2x+y-2z=-2; x+y-3z=-6

TURN OVER

Q5. (a) Attempt any two (4 marks each) **[8]**

(i) Find the particular solution of: $(D^2+D-2)y=0$, when x=0, y=1 and $\frac{dy}{dx}=0$

(ii) Solve the differential equation: $(1-x)dy-(1+y)dx=0$

(iii) Solve: $(D^2+3D+2)y=x+x^2$

(b) Attempt any one (3 marks) **[3]**

(i) Form the differential equation for $x^2 + y^2 - 2ax = 10$

(ii) Solve $(1-x)dy - (1+y)dx = 0$. Also find the particular solution, if y = 2 when x = 1

Q6. (a) Attempt any two (4 marks each) **[8]**

(i) Use Lagrange's Interpolation formula to find the polynomial passing through the points (0,8),(1,4) & (3,2). Hence find the value of y when x=2.

(ii) Evaluate $\int_0^2 x^2 dx$ by using Trapezoidal rule (with h=0.2)

(iii) Estimate the missing value by using E and Δ from the following:

x	1	2	3	4	5
y	2	4	8	-	32

(b) Attempt any one (3 marks) **[3]**

(i) For a certain function f(x), f(1) = 10, f(2)=16, f(3) =26 and f(4) = 40 , estimate f(2.5) by Newton's forward difference formula.

(ii) Solve: $\left(\frac{\Delta^2}{E}\right)x^4$ by taking h =1